Math 309 - Worksheet - Linear algebra review

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix}, C = \begin{bmatrix} 3 & i \\ -i & 1 \end{bmatrix}, F = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 2 & -1 & 3 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

1. **Determinant:** Compute $\det B, \det C, \det F$.

2. **Linear systems with invertible matrix:**

   Consider the equation $Cx = v$ where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

   Find the solution to $Cx = v$.

   Find the solution to $Cx = 0$ (homogeneous equation).

3. **Linear systems with noninvertible matrix:**
   Find the set of solutions to $Bx = 0$.

   Find the set of solutions to $Bx = v$.

   Find the set of solutions to $Bx = \begin{bmatrix} 1 \\ -i \end{bmatrix}$.

4. **Linear systems with noninvertible matrix cont'd:**

Find the set of solutions to $Fx = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

5. **Linear (in)dependence:**
Are the three column vectors of F linearly independent? If they are linearly dependent, find a linear relation among them.

6. **Eigenvalues and eigenvectors:**

Given a matrix M, if $Mx = \lambda x$, equivalently $(M - \lambda I)x = 0$, for some $x \neq 0$, then λ is an **eigenvalue** of M and x is an **eigenvector** of M corresponding to λ.

Note that there exists $x \neq 0$ such that $(M - \lambda I)x = 0$ if and only if $\det(M - \lambda I) = 0$.

(a) Find all the eigenvalues of the matrix A and the set of eigenvectors corresponding to each eigenvalue.

(b) Write $A = PDP^{-1}$ where D is a diagonal matrix with the diagonal entries being the eigenvalues.