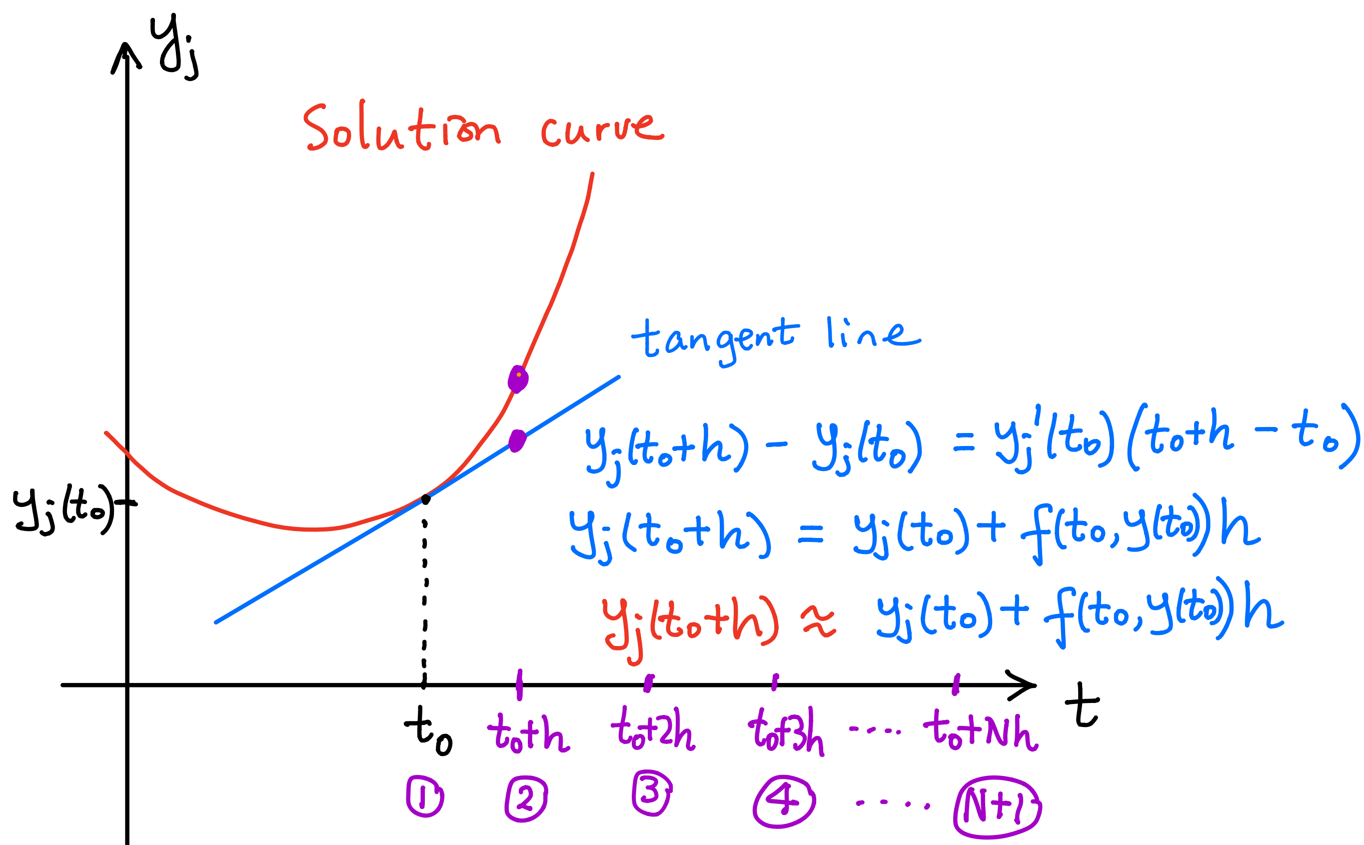
Euler's numerical method

1st order system

$$\begin{bmatrix} y_1'(t) \\ \vdots \\ y_k'(t) \end{bmatrix} = \begin{bmatrix} f_1(t, \overset{(y_1,\dots,y_k)}{\overset{\shortparallel}{y}}) \\ \vdots \\ f_k(t, y) \end{bmatrix}, \quad y(t_0) = y_0 = \begin{bmatrix} y_1(t_0) \\ \vdots \\ y_k(t_0) \end{bmatrix}$$



1	2	3
$y(t_0) = \begin{bmatrix} y_1(t_0) \\ \vdots \\ y_k(t_0) \end{bmatrix}$ $= y_{1}$	$y(t_0+h)$ $\approx y(t_0) + f(t_0, y(t_0))h$ $= y_{2}$	$y(t_0+2h)$ $\approx y(t_0+h) + f(t_0+h, y(t_0+h))h$ $\approx y_{2} + f(t_0+h, y_{2})h$ $= y_{3}$

$$\cdots \quad \begin{array}{l} N+1 \\ y(t_0+Nh) \\ \approx y(t_0+(N-1)h) + f(t_0+(N-1)h,\, y(t_0+(N-1)h))\,h \\ = y_{N+1} \end{array}$$

For $n = 1, \ldots, N$

$$y_{n+1} = y_n + f(t_n, y_n)\,h$$

HW 1 #9 (Pendulum)

a) $\begin{bmatrix} \theta' \\ \omega' \end{bmatrix} = \begin{bmatrix} \omega \\ -\frac{g}{L}\sin\theta \end{bmatrix}$, $\begin{bmatrix} \theta(0) \\ \omega(0) \end{bmatrix} = \begin{bmatrix} \frac{\pi}{4} \\ 0 \end{bmatrix}$

Say $\frac{g}{L} = 1$

$\Rightarrow \begin{bmatrix} \theta' \\ \omega' \end{bmatrix} = \begin{bmatrix} \omega \\ -\sin\theta \end{bmatrix} = f$, $\quad y = \begin{bmatrix} \theta(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} y[1] \\ y[2] \end{bmatrix}$

use euler.jl to solve this system

b) Linearized eqn $\theta'' = -\frac{g}{L}\theta$ with $\theta(0) = \frac{\pi}{4}$, $\theta'(0) = 0$

has solution $\theta(t) = \frac{\pi}{4}\cos t$

use euler.jl to compare this to the solution for the nonlinear equation with this initial condition.