Intro. to system of differential eqn's Cont'd

higher order eqn's $\Longleftrightarrow$ a system of 1st order eqn's

Ex Write $mu''(t) + \gamma u'(t) + k u(t) = F(t)$ as a system of 1st order eqn's

Let $u_1(t) = u(t)$

$u_2(t) = u_1'(t) = u'(t)$

$\Rightarrow m u_2' + \gamma u_2 + k u_1 = F(t)$

$$\Rightarrow \begin{cases} u_1' = u_2 \\ u_2' = -\frac{k}{m} u_1 - \frac{\gamma}{m} u_2 + \frac{F(t)}{m} \end{cases}$$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\gamma}{m} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{F}{m} \end{bmatrix}$$

Ex $u^{(4)} - u = 1$

$u_1 = u$

$u_2 = u_1' = u'$

$u_3 = u_2' = u''$

$u_4 = u_3' = u'''$

$\Rightarrow u_4' - u_1 = 1$

$$\begin{cases} u_1' = u_2 \\ u_2' = u_3 \\ u_3' = u_4 \\ u_4' = u_1 + 1 \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Ex: Write the system $\begin{cases} x_1' = x_1 + x_2 \ \textcircled{1} \\ x_2' = -2x_1 \ \textcircled{2} \end{cases}$, $\begin{matrix} x_1(0) = 0 \\ x_2(0) = 0 \end{matrix}$

as a single 2nd order eqn in x.

Let $\boxed{x = x_1}$

$\textcircled{1} \Rightarrow \quad x' = x + x_2 \qquad \Rightarrow \boxed{x_2 = x' - x}$

$\textcircled{2} \Rightarrow \quad (x'-x)' = -2x$

$$\boxed{x'' - x' + 2x = 0}$$

$x(0) = x_1(0) = 0$

$x'(0) = x_2(0) + x(0) = 0$