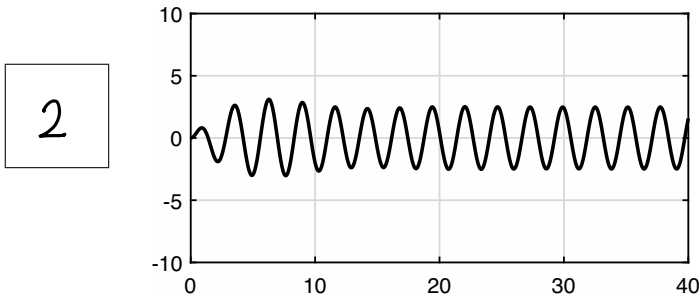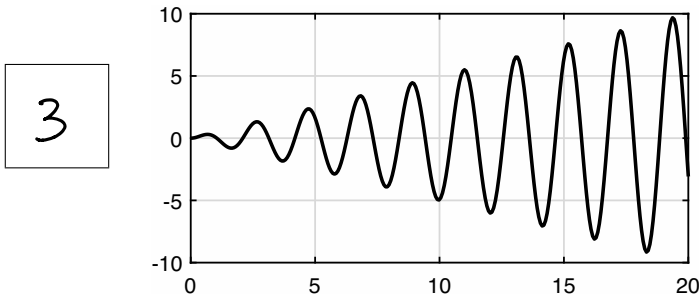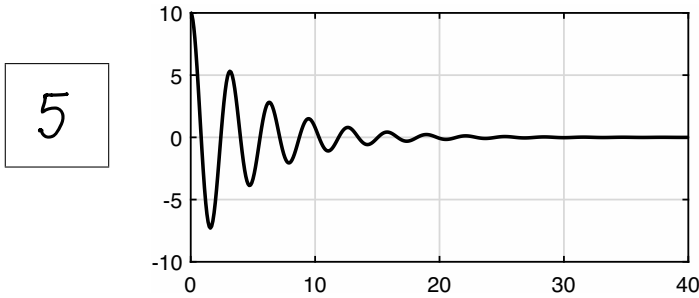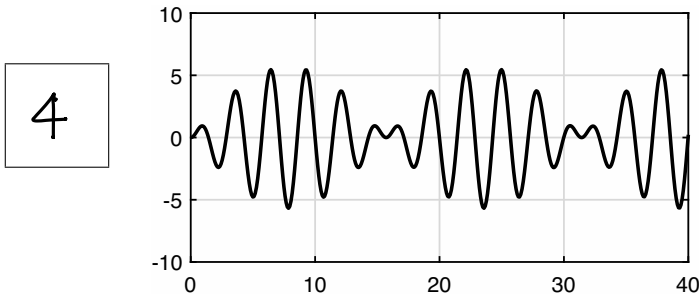4 (20 points)

Match the differential equations to the plots of their soutions. Write the number of the correct differential equation in the box to the left of the plot. *Briefly explain your answer!*

4

5

3

2

1
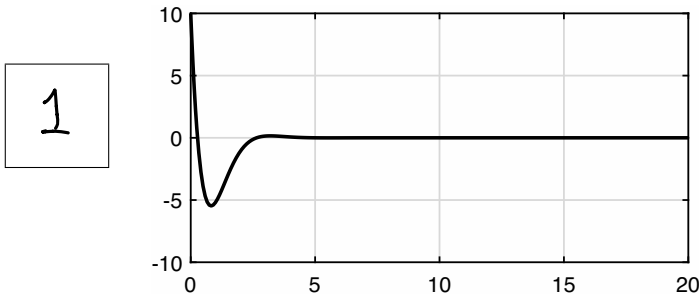
(1) $y'' + 3y' - 4y = 0$

char eq: $r^2 + 3r - 4 = 0$

$(r+4)(r-1) = 0$

2 real roots $\Rightarrow$ overdamped

decay without

oscillation

(2) $y'' + 0.4y' + 4y = 5\cos 2.4t$

Forced vibration with

damping $\Rightarrow$ eventually vibrates

with constant amplitude

(3) $y'' + 9y = 3\cos 3t$

char eq: $r^2 + 9 = 0$

$r = \pm 3i$

natural freq = 3

driving freq = 3

$\Rightarrow$ resonance

(4) $y'' + 4y = 5\cos 2.4t$

$r^2 + 4 = 0$

$r = \pm 2i$

natural freq = 2

$\neq$

driving freq = 2.4

$\Rightarrow$ Beats

(5) $y'' + 0.4y' + 4y = 0$

Last one, can just use elimination of choice

or: char eq: $r^2 + 0.4r + 4$

$r = \dfrac{-0.4 \pm \sqrt{(0.4)^2 - 16}}{2}$

CX roots $\Rightarrow$ underdamped

decay with oscillation