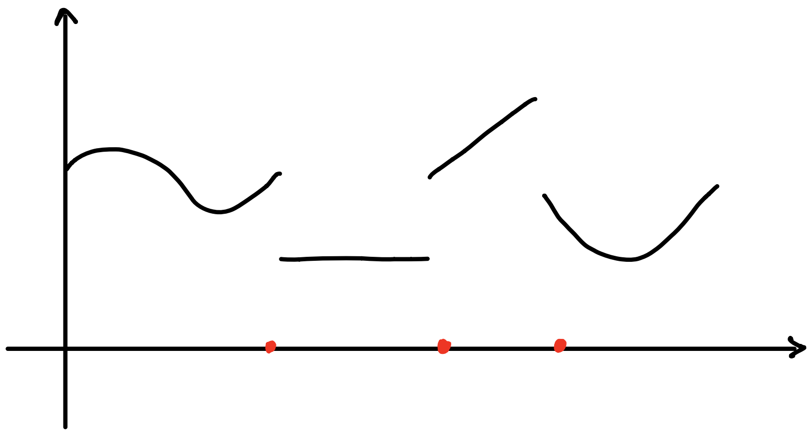
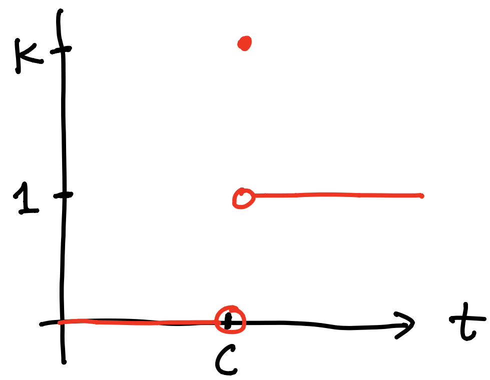
piecewise continuous functions



Jump discont.

Ex 0

$$f(t) = \begin{cases} 0, & 0 \le t < c \\ k, & t = c \\ 1, & t > c \end{cases}$$



$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t) e^{-st}\, dt$$

$$= \int_c^\infty e^{-st}\, dt = -\frac{e^{-st}}{s}\Big|_c^\infty = \frac{e^{-cs}}{s}$$

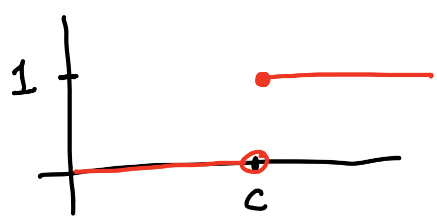
$s > 0$

doesn't depend on k (value at a single pt)

$\Rightarrow$ no unique $\mathcal{L}^{-1}$

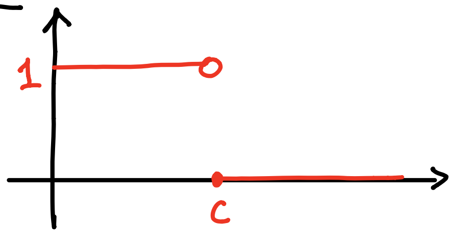
Def^n: unit step function

$$u_c(t) = \begin{cases} 0, & 0 \le t < c \\ 1, & t \ge c \end{cases}$$



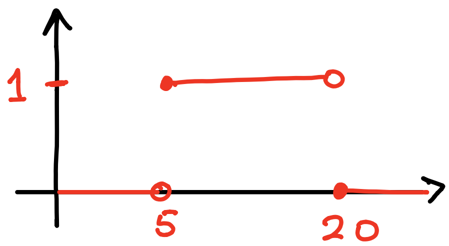
$$\mathcal{L}\{u_c(t)\} = \frac{e^{-cs}}{s}, \quad s > 0$$

Ex 1



$$1 - u_c(t)$$

Ex 2



$$g(t) = u_5(t) - u_{20}(t)$$

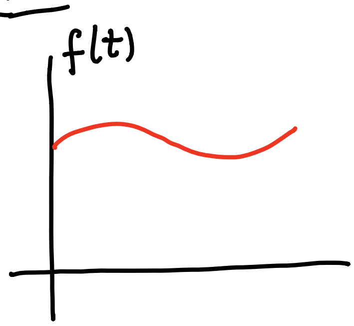
$$\mathcal{L}\{g(t)\} = \mathcal{L}\{u_5(t)\} - \mathcal{L}\{u_{20}(t)\}$$

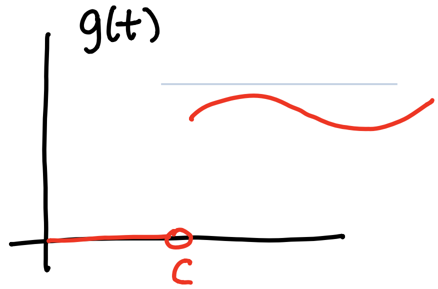
$$= \frac{e^{-5s}}{s} - \frac{e^{-20s}}{s}$$

Ex 3

$$g(t) = \begin{cases} 2, & 0 \le t < 4 \\ 5, & 4 \le t < 7 \\ -1, & 7 \le t < 9 \\ 1, & t \ge 9 \end{cases} = 2 + 3u_4(t) - 6u_7(t) + 2u_9(t)$$

Ex 4



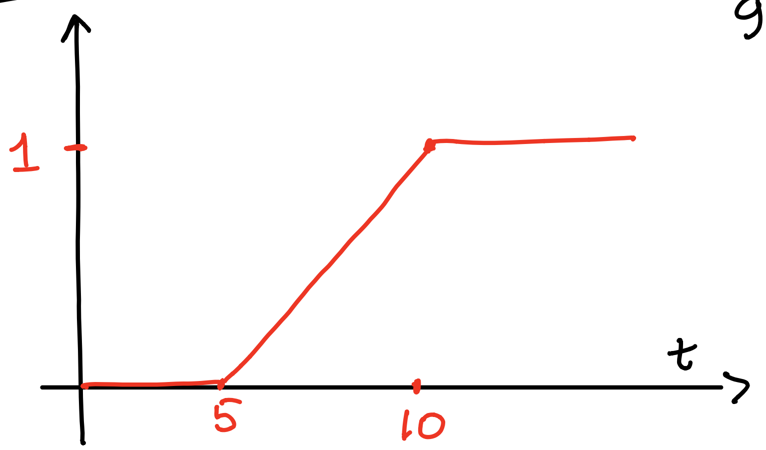


translation

$$g(t) = \begin{cases} 0, & t < c \\ f(t-c), & t \ge c \end{cases}$$

$$= u_c(t) f(t-c)$$

Ex 5



$$g(t) = \begin{cases} 0, & 0 \le t \le 5 \\ \frac{t-5}{5}, & 5 \le t \le 10 \\ 1, & t \ge 10 \end{cases}$$

$$= u_5(t)\left(\frac{t-5}{5}\right) - u_{10}(t)\left(\frac{t-10}{5}\right)$$

$$= u_5(t) f(t-5) - u_{10}(t) f(t-10)$$

$$f(t) = \frac{t}{5}$$

Property 4 If $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a$,

then $\mathcal{L}\{u_c(t) f(t-c)\} = \int_0^\infty u_c(t) f(t-c) e^{-st}\, dt$

$$= \int_c^\infty f(t-c)\, e^{-st}\, dt$$

$$(\tau = t-c) \quad = \int_0^\infty f(\tau)\, e^{-s(\tau+c)}\, d\tau$$

$$= e^{-cs} \int_0^\infty f(\tau)\, e^{-s\tau}\, d\tau$$

$$\boxed{\mathcal{L}\{u_c(t) f(t-c)\} = e^{-cs} F(s)}, \quad s > a.$$

Ex 5 (cont'd) $\mathcal{L}\{g(t)\} = e^{-5s}\mathcal{L}\{\frac{t}{5}\} - e^{-10s}\mathcal{L}\{\frac{t}{5}\}$

$$= \frac{1}{5}\left(e^{-5s} - e^{-10s}\right)\mathcal{L}\{t\}$$

$$= \frac{1}{5s^2}\left(e^{-5s} - e^{-10s}\right)$$

Property 5 If $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a$,

then $\mathcal{L}\{e^{ct} f(t)\} = \int_0^\infty e^{ct} f(t) e^{-st} dt$

$$= \int_0^\infty f(t) e^{-(s-c)t} dt$$

$$\boxed{\mathcal{L}\{e^{ct} f(t)\} = F(s-c)} \quad , \qquad s - c > a$$

i.e. $s > a + c$