§2.2. Separable eqn's

General 1st order eqn: $\frac{dy}{dx} = f(x, y)$

Notation change: indept var = x (instead of t)

Separable eqn: $\frac{dy}{dx} = \frac{M(x)}{N(y)}$

$$\int N(y)\,dy = \int M(x)\,dx$$

Ex: ① $\frac{dy}{dx} = xy$ (separable)

$$\frac{dy}{y} = x\,dx$$

② $\frac{dy}{dx} = x + y$ (not separable)

Ex: $\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}$, $y(0) = -1$

$$\int 2(y-1)\,dy = \int (3x^2 + 4x + 2)\,dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + A$$

$$y^2 - 2y - (x^3 + 2x^2 + 2x + A) = 0$$ (Implicit general solution)

$y(0) = -1 \Rightarrow (-1)^2 - 2(-1) - (0 + A) = 0$

$\Rightarrow A = 3$

$y^2 - 2y - (x^3 + 2x^2 + 2x + 3) = 0$ (Implicit soln to the IVP)

Recall quadratic formula:

$y^2 + by + c = 0$, $y = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$

$y(x) = 1 \pm \sqrt{x^3 + 2x^2 + 2x + A + 1}$ (Explicit general solution)

$y(0) = -1$ $\Rightarrow$ $y(x) = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$ (Explicit soln to IVP)

$= 1 - \sqrt{(x+2)(x^2+2)}$

$= 0$ when $x = -2$

$y = 1$

y' vertical at $y = 1$